MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2015/2016

DIM5068 – MATHEMATICAL TECHNIQUES 2

(for diploma students only)

11 MARCH 2016
9.00 am – 11.00 am
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of 3 pages excluding cover page and appendix.

2. Attempt ALL **FIVE** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the Answer Booklet provided.

4. Key formulae are given in the Appendix.

Please answer ALL questions and show the necessary working steps. Each question is 20 marks.

Question 1

a. Simplify $\frac{5-i}{3-2i}$ and represent the answer in standard form, $a + bi$. (4 marks)

b. Given the complex number $z = -\sqrt{5} - 2i$. Write an expression for z in polar form. (5 marks)

c. Evaluate the following limits.

i. $\lim_{u \to 25} \frac{\sqrt{u} - 5}{u - 25}$. (4 marks)

ii. $\lim_{x \to \infty} \frac{6x^3 - 5x + 2}{16 + 2x^3}$. (4 marks)

iii. Given that; $\lim_{x \to 3} f(x) = \frac{4}{5}$, $\lim_{x \to 3} g(x) = m$ and $\lim_{x \to 3} h(x) = \frac{3}{4}$. Find the value of m if $\lim_{x \to 3} \frac{5f(x)h(x)}{g(x)} = \frac{3}{10}$. (3 marks)

[TOTAL 20 MARKS]

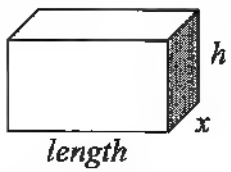
Question 2

a. Find the derivatives of the following functions.

i. $y = 3x^2 - 2xe^{4x}$ (4 marks)

ii. $y = \ln(\sqrt{x})$ (4 marks)

b. Ahmed would like to make a cuboid using a piece of wire. The requirement needed for the cuboid is the length must be 3 times its width, x, as shown in the diagram below.



i. Given the total length of the wire is 192cm. Express the height of the cuboid, h, in terms of x. (3 marks)

ii. What is the width of the box, x, in order to maximize its volume? (6 marks)

iii. Calculate the maximum volume of the box. (2 marks)

[TOTAL 20 MARKS]

Continued.....

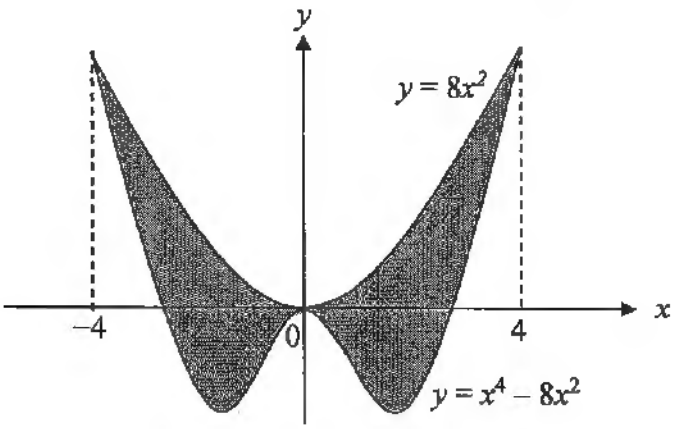
Question 3

a. Find the following integrals.

i. $\int \frac{5}{s} + \frac{4}{s^3} - 12e^s - \sqrt{3}\ ds$. (4 marks)

ii. $\int_{-10^\circ}^{35^\circ} \sec^2(10^\circ + x)\, dx$. [Hint: use **Substitution Rule**] (6 marks)

b. Find the shaded area enclosed by $y = 8x^2$ and $y = x^4 - 8x^2$ from $x = -4$ to $x = 4$. (10 marks)



[TOTAL 20 MARKS]

Question 4

a. Solve the differential equation, $3t\frac{dy}{dt} = 2yt^3 - 9ty$. (4 marks)

b. Given the differential equation, $x^2\frac{dy}{dx} - 3xy = x^6e^x$.

i. Identify $P(x)$ and $Q(x)$. (2 marks)

ii. From part b(i), determine the integrating factor $\mu(x)$. (3 marks)

iii. Then, show that $y = x^3(2xe^x - 2e^x + c)$ [Hint: use **Integration by Part**]. (7 marks)

c. Find the **general solution** for the differential equation, $2y'' - 11y' + 12y = 0$. (4 marks)

[TOTAL 20 MARKS]

Continued.....

Question 5

a. Find the angle between $\mathbf{a} = \langle 1,-2,-1 \rangle$ and $\mathbf{b} = \langle -6,2,-3 \rangle$. (6 marks)

b. Given that $P = (1,0,2)$, $Q = (-1,2,3)$ and $R = (3,4,5)$.

i. Find the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$. (3 marks)

ii. Find the area of the triangle PQR. (6 marks)

c. Find the **parametric equation** and **symmetric equation** for the line through the points (2,1,5) and (7,3,2). (5 marks)

[TOTAL 20 MARKS]

End of page.

APPENDIX

Derivatives: $f'(a)=\lim_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$

Differentiation Rules

General Formulae

1. $\dfrac{d}{dx}[f(x)g(x)]=f(x)g'(x)+g(x)f'(x)$
2. $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right]=\dfrac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$
3. $\dfrac{d}{dx}(x^n)=nx^{n-1}$
4. $\dfrac{d}{dx}[g(x)]^n=n[g(x)]^{n-1}\cdot g'(x)$

Exponential and Logarithmic Functions

1. $\dfrac{d}{dx}(e^x)=e^x$
2. $\dfrac{d}{dx}(a^x)=a^x\ln a$
3. $\dfrac{d}{dx}(\ln x)=\dfrac{1}{x}$
4. $\dfrac{d}{dx}(\log_a x)=\dfrac{1}{x\ln a}$

Trigonometric Functions

1. $\dfrac{d}{dx}(\sin x)=\cos x$
2. $\dfrac{d}{dx}(\cos x)=-\sin x$
3. $\dfrac{d}{dx}(\tan x)=\sec^2 x$
4. $\dfrac{d}{dx}(\csc x)=-\csc x\cot x$
5. $\dfrac{d}{dx}(\sec x)=\sec x\tan x$
6. $\dfrac{d}{dx}(\cot x)=-\csc^2 x$

Table of Integrals

1. $\int u\,dv=uv-\int v\,du$
2. $\int u^n\,du=\dfrac{u^{n+1}}{n+1}+C,\quad n\neq -1$
3. $\int\dfrac{du}{u}=\ln|u|+C$
4. $\int e^u du=e^u+C$
5. $\int\sin u\,du=-\cos u+C$
6. $\int\cos u\,du=\sin u+C$
7. $\int\sec^2 u\,du=\tan u+C$
8. $\int\csc^2 u\,du=-\cot u+C$
9. $\int\sec u\tan u\,du=\sec u+C$
10. $\int\csc u\cot u\,du=-\csc u+C$

Application of Integrals:

Areas between Curve, $A=\int_a^b[f(x)-g(x)]\,dx$

Differential Equations

Linear Differential Equations

$$\frac{dy}{dx} + p(x)y = q(x) \quad \Rightarrow \quad \mu y = \int \mu q(x)\,dx, \text{ where } \mu = e^{\int p(x)dx}$$

Constant Coefficient of Homogeneous Equations

Roots of Auxiliary Equation, $r = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

General Solutions to the Auxiliary Equation:

2 Real & Unequal Roots ($b^2 - 4ac > 0$)	$y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$
Repeated Roots ($b^2 - 4ac = 0$)	$y = c_1 e^{rx} + c_2 x e^{rx}$
2 Complex Roots ($b^2 - 4ac < 0$)	$y = e^{ax}(c_1 \cos bx + c_2 \sin bx)$

Constant Coefficient of Non-Homogeneous Equations

$y = y_c + y_p$ [y_c: complementary solution, y_p: particular solution]

Vector

Length of Vector

The length of the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ is $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

Dot Product

If θ is the angle between the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = |\mathbf{a}||\mathbf{b}| \cos\theta$

Cross Product

If θ is the angle between the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$\mathbf{a} \times \mathbf{b} = \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle$

$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin\theta$

Area for parallelogram *PQRS*

$$= \left|\vec{PQ} \times \vec{PR}\right|$$

Area for triangle PQR

$$= \frac{1}{2}\left|\vec{PQ} \times \vec{PR}\right|$$

Equation of Lines

Vector equation: $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$

Parametric equations: $x = x_0 + at \quad y = y_0 + bt \quad z = z_0 + ct$

Symmetric equation: $\dfrac{x - x_0}{a} = \dfrac{y - y_0}{b} = \dfrac{z - z_0}{c}$

Equation of Planes

Vector equation: $\mathbf{n} \cdot \mathbf{r} = \mathbf{n} \cdot \mathbf{r}_0$

Scalar equations: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

Linear equation: $ax + by + cz + d = 0$

Angle between Two Planes: $\cos\theta = \dfrac{\mathbf{n}_1 \bullet \mathbf{n}_2}{|\mathbf{n}_1||\mathbf{n}_2|}$